Gravitational-Wave Astronomy

1060-711: Astronomical Observational Techniques and Instrumentation

Guest Lecturer: Prof. John T. Whelan

2013 May 1

References

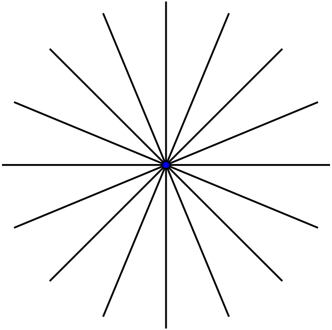
- Creighton & Anderson, *Gravitational-Wave Physics and Astronomy* (Wiley, 2011). ISBN 978-3-527-40886-3
- Maggiore, *Gravitational Waves: Volume 1: Theory and Experiments* (Oxford, 2007). ISBN 978-0-198-57074-5
- Saulson, *Fundamentals of Interferometric Gravitational Wave Detectors* (World Scientific, 1994). ISBN 978-9-810-21820-1

Outline

Outline

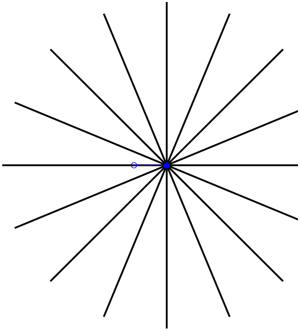
Action at a Distance

- Newtonian gravity: mass generates gravitational field
- Lines of force point towards object



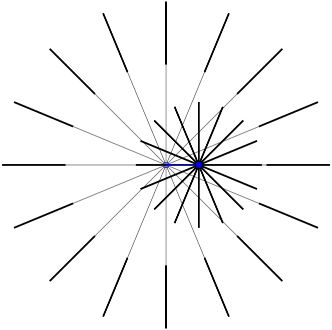
Issues with Causality

- Move object; Newton says: lines point to new location
- Relativity says: can't communicate faster than light to avoid paradoxes
- You could send me supraluminal messages via grav field



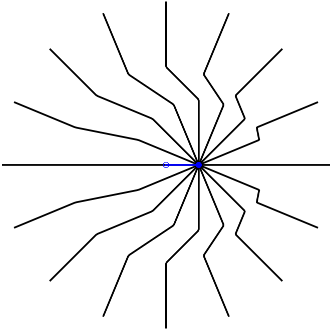
Gravitational Speed Limit

- If I'm 10 light years away,
  I can't know you moved
  the object 6 years ago
- Far away, gravitational field lines
  have to point to
  old location of the object



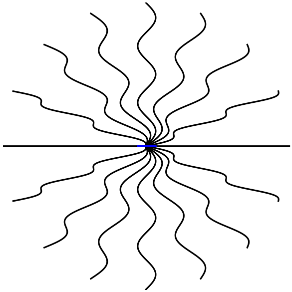
Gravitational Shock Wave

- Sudden motion (acceleration) of object generates gravitational shock wave expanding at speed of light



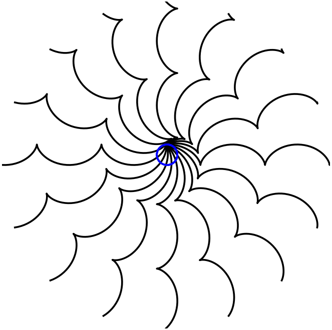
Ripples in the Gravitational Field

- Move object back & forth
  $\longrightarrow$ gravitational wave
- Same argument applies to electricity:
  - can derive magnetism as relativistic effect
  - accelerating charges generate electromagnetic waves propagating @ speed of light

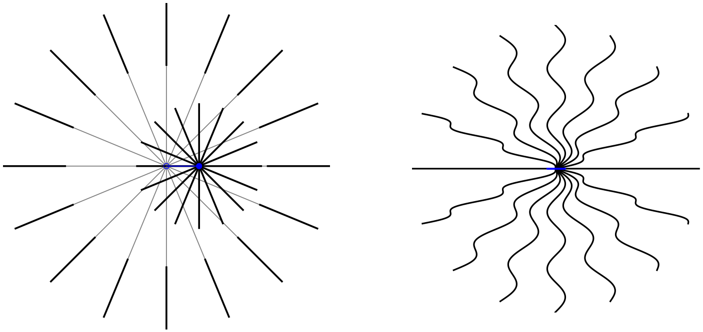


Gravitational Wave from Orbiting Mass?

- Move around in a circle
- Still get grav wave pattern, but looks a bit funny
- Time to move beyond simple pseudo-Newtonian picture



Gravity + Causality = Gravitational Waves



- In Newtonian gravity, force dep on distance btwn objects
- If massive object suddenly moved, grav field at a distance would change instantaneously
- In relativity, no signal can travel faster than light
  $\longrightarrow$ time-dep grav fields must propagate like light waves

Gravity as Geometry

- Minkowski Spacetime:

$$ds^2 = -c^2(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2$$

$$= \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}^{\mathrm{tr}} \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix} = \eta_{\mu\nu} dx^\mu dx^\nu$$

- General Spacetime:

$$ds^2 = \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\mathrm{tr}} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = g_{\mu\nu} dx^\mu dx^\nu$$

Gravitational Wave as Metric Perturbation

- For GW propagation & detection, work to 1st order in $h_{\mu\nu} \equiv$ difference btwn actual metric $g_{\mu\nu}$ & flat metric $\eta_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

  ($h_{\mu\nu}$ "small" in weak-field regime, e.g. for GW detection)
- Convenient choice of gauge is transverse-traceless:

$$h_{0\mu} = h_{\mu 0} = 0 \qquad \eta^{\nu\lambda} \frac{\partial h_{\mu\nu}}{\partial x^\lambda} = 0 \qquad \eta^{\mu\nu} h_{\mu\nu} = \delta^{ij} h_{ij} = 0$$

  In this gauge:
  - Test particles w/constant coörds are freely falling
  - Vacuum Einstein eqns $\Longrightarrow$ wave equation for $\{h_{ij}\}$:

$$\left(-\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \nabla^2 \right) h_{ij} = 0$$

Gravitational Wave Polarization States

- Far from source, GW looks like plane wave prop along $\vec{k}$
  TT conditions mean, in convenient basis,

$$\{k_i\} \equiv \boldsymbol{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \{h_{ij}\} \equiv \mathbf{h} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

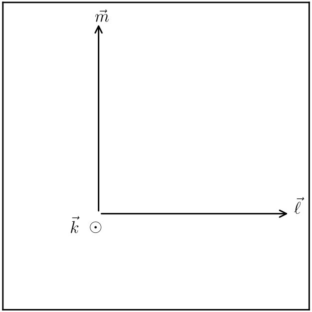
  where $h_+\left(t - \frac{x^3}{c}\right)$ and $h_\times\left(t - \frac{x^3}{c}\right)$ are components in "plus" and "cross" polarization states
- More generally

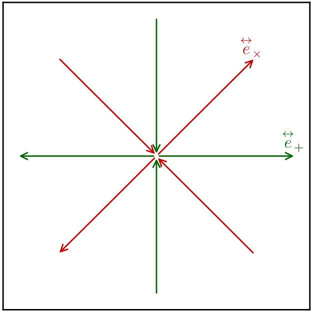
$$\overleftrightarrow{h} = h_+\left(t - \frac{\vec{k}\cdot\vec{r}}{c}\right)\overleftrightarrow{e}_+ + h_\times\left(t - \frac{\vec{k}\cdot\vec{r}}{c}\right)\overleftrightarrow{e}_\times$$

The Polarization Basis

- wave propagating along $\vec{k}$;
  construct $\overleftrightarrow{e}_{+,\times}$ from $\perp$ unit vectors $\vec{\ell}$ & $\vec{m}$:

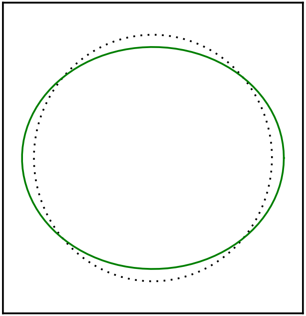
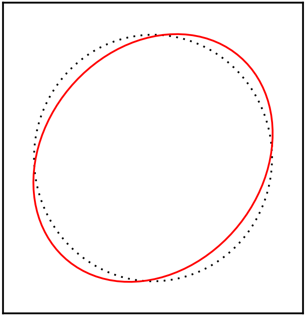
$$\overleftrightarrow{e}_{+} = \vec{\ell} \otimes \vec{\ell} - \vec{m} \otimes \vec{m} \qquad \overleftrightarrow{e}_{\times} = \vec{\ell} \otimes \vec{m} + \vec{m} \otimes \vec{\ell}$$





Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:

Plus (+) Polarization	Cross (×) Polarization
	

Multipole Expansion for Gravitational Radiation

- **"Electric Dipole"?**
  No, "dipole moment" $\int \vec{r}\, dm \propto$ ctr of mass
  COM can't oscillate (also no negative "charge" in GR)
- **"Magnetic Dipole"?** No, "mag moment"
  $\frac{1}{2}\int \vec{r} \times \vec{v}\, dm \propto$ spin, another conserved quantity
- **"Electric Quadrupole"?** Yes! In TT gauge,

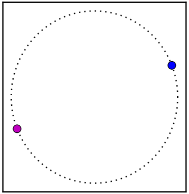
$$h_{ij}(t) = \frac{2G}{c^4 d} P^{\mathrm{TT}\vec{k}\,k\ell}_{ij} \ddot{I\!\!\!-}_{k\ell}(t - d/c)$$

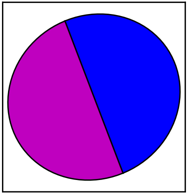
  in terms of mass quadrupole moment

$$I\!\!\!-_{ij} = \int \left(r_i r_j - \delta_{ij} \frac{r^2}{3} \right) dm$$

Quadrupole Radiation From Rotating/Orbiting System

- Equatorial moments of inertia I_1, I_2
- Orbital/rotational ang vel Ω
- GW frequency $f_{\mathrm{gw}} = 2\frac{\Omega}{2\pi}$





- Since $\ddot{I\!\!\!-} \sim (2\Omega)^2 \, |I_1 - I_2|$,

$$\overleftrightarrow{h} = \frac{4G\Omega^2(I_1 - I_2)}{c^4 d} \left(\overleftrightarrow{e}_+ \frac{1+\cos^2\iota}{2} \cos 2\Omega t + \overleftrightarrow{e}_\times \cos\iota \sin 2\Omega t \right)$$

- For binary system w/masses m_1, m_2 and separation r,

$$I_1 = 0 \qquad \text{and} \qquad I_2 = \mu r^2$$

where $\mu = \frac{m_1 m_2}{m_1+m_2} = \frac{m_1 m_2}{M}$ is the reduced mass

Radiation from Quasicircular Binary

Total mass $M = m_1 + m_2$; reduced mass $\mu = \frac{m_1 m_2}{M}$; orbital freq Ω

- Amplitude is $h_0 = \frac{4G\Omega^2\mu r^2}{c^4 d}$
- Kepler's 3rd law: $GM = r^3\Omega^2 \implies r^2 = (GM\Omega^{-2})^{2/3}$
  $h_0 = \frac{4G^{5/3}M^{2/3}\mu\Omega^{2/3}}{c^4 d} = \frac{4(G\mathcal{M}_c)^{5/3}\Omega^{2/3}}{c^4 d}$
  where $\mathcal{M}_c = M\eta^{3/5}$ is chirp mass & $\eta = \frac{\mu}{M}$ is symm mass ratio
- Orbit will evolve due to GW emission (radiation reaction): energy lost, r dec., Ω inc., h_0 inc.: "chirp"
- Quasicircular assumption breaks down when $r_{\text{isco}} \approx 6GM/c^2$ near "innermost stable circular orbit" (ISCO); orbital freq @ ISCO is $\Omega_{\text{isco}} \approx \sqrt{\frac{GM}{r_{\text{isco}}^3}} = \frac{c^3}{6^{3/2}GM}$
- Modelling final merger accurately requires numerical simulations like those done in RIT CCRG

Some Sources of Gravitational Waves

Band: ground, space, pulsar timing

- Binary coalescence (inspiral+merger+ringdown):
  - Supermassive BH binary
  - extreme mass ratio (stellar mass + SMBH)
  - Stellar mass BH and/or neutron star
- Galactic white dwarf binary orbit (continuous source)
- Rotating neutron star (pulsar, LMXB, etc)
- Supernova, SGR
- Cosmological background (primordial, phase transitions, cosmic superstrings, etc)
- SMBH flyby

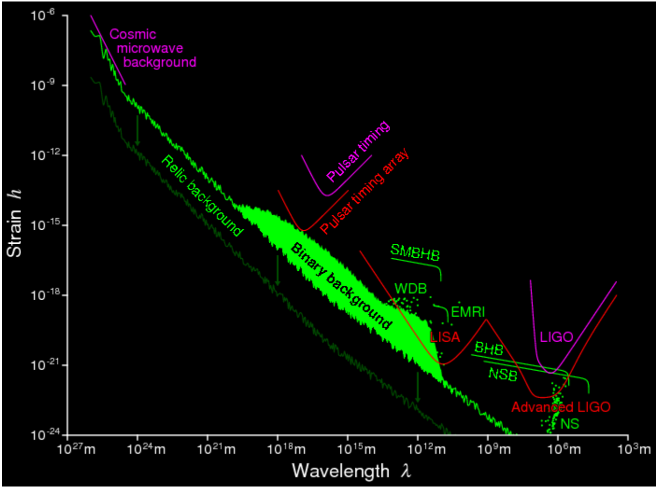
Outline

Methods for Measuring Gravitational Waves

- Cosmic Microwave Background Perturbations ($f_{gw} \sim H_0 \sim 10^{-18}$ Hz)
- Pulsar Timing Arrays (10^{-9} Hz $\lesssim f_{gw} \lesssim 10^{-7}$ Hz)
- Laser Interferometers
  - Space-Based (10^{-3} Hz $\lesssim f_{gw} \lesssim 10^{-1}$ Hz)
  - Ground-Based (10^{1} Hz $\lesssim f_{gw} \lesssim 10^{3}$ Hz)
- Resonant-Mass Detectors (narrowband, $f_{gw} \sim 10^{3}$ Hz)

Note, observable GW freq cover 20 orders of magnitude, similar to EM radiation, but the frequencies are much lower (10^{3} Hz $\lesssim f_{em} \lesssim 10^{23}$ Hz)

The Gravitational-Wave Spectrum



http://www.tapir.caltech.edu/∼teviet/Waves/

Rogues' Gallery of Ground-Based Interferometers

LIGO Hanford (Wash.)

LIGO Livingston (La.)

GEO-600 (Germany)

Virgo (Italy)

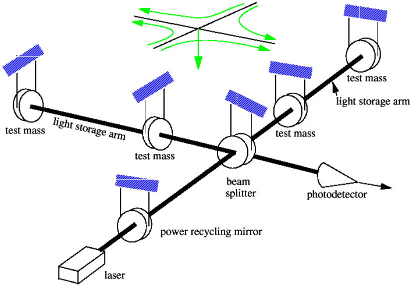
Initial Gravitational Wave Detector Network

- “1st generation” ground-based interferometertic GW detectors (kilometer scale):
  - TAMA 300 (Tokyo, Japan) first online, late 90s; now offline
  - LSC detectors conducting science runs since 2002
    - LIGO Hanford (4km H1 & 2km H2)
    - LIGO Livingston (4km L1)
    - GEO-600 (600m G1)
  - Virgo (3km V1) started science runs in 2007
  - LSC-Virgo long joint runs @ design sensitivity 2005-2010
- LIGO and Virgo being upgraded to 2nd generation “advanced” detectors ($10\times$ improvement in sensitivity)
- GEO-600 remains operational in “astrowatch” mode in case there’s a nearby supernova

Advanced Gravitational Wave Detector Network

- “2nd generation” ground-based interferometric GW detectors:
  - Adv LIGO expected to take science data from 2014 or 2015
    4km detectors in Livingston, La. & Hanford, Wa.
  - Advanced Virgo should be on comparible timescale
  - KAGRA (cryogenic detector in Kamioka mine, Japan)
    uses 2.5-generation technology
  - Third advanced LIGO detector (4km)
    may be installed in India, taking data c.2019+
    Big payoff for sky localization via tringulation
- Planning for 3rd generation already underway:
  - Einstein Telescope in Europe
  - USA 3G plans still under development
    (RIT CCRG involved in astrophysics planning)

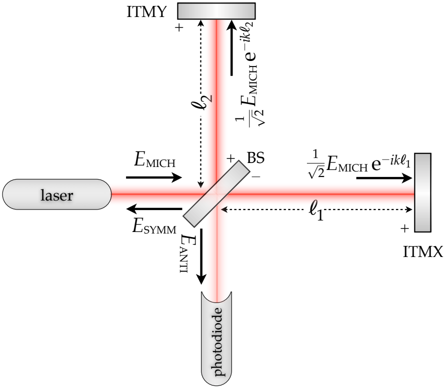
Experimental Details: LIGO



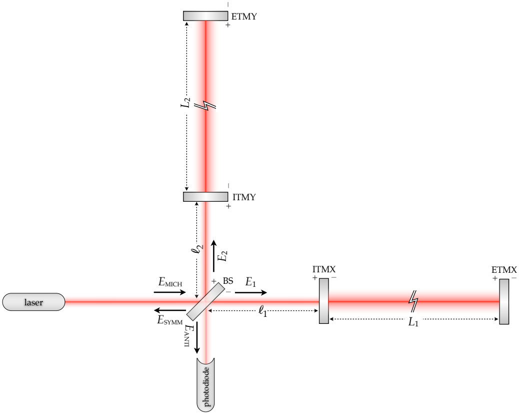
- Initial/enhanced LIGO was a power-recycled Fabry-Pérot Michelson interferometer
- Advanced LIGO will be a dual-recycled Fabry-Pérot Michelson interferometer
- Basic idea: use interferometry to measure changes in difference of arm lengths to detect $h \lesssim 10^{-20}$

Michelson Interferometer

w/$\lambda_{\text{laser}} \sim 10^{-6}\,\text{m}$ & $L \sim 10^{3}\,\text{m}$
would need to measure
$\delta L \sim 10^{-11}\,\lambda_{\text{laser}}$
to detect $h \sim 10^{-20}$

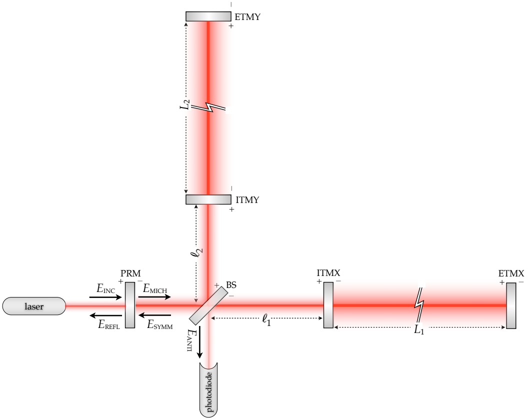


Fabry-Pérot Cavities



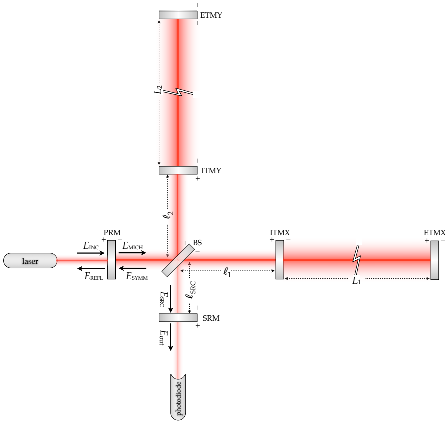
Increase "effective length" of arms by keeping light in resonance within FP cavities; finesse $\sim$ 200 amplifies signal

Power Recycling



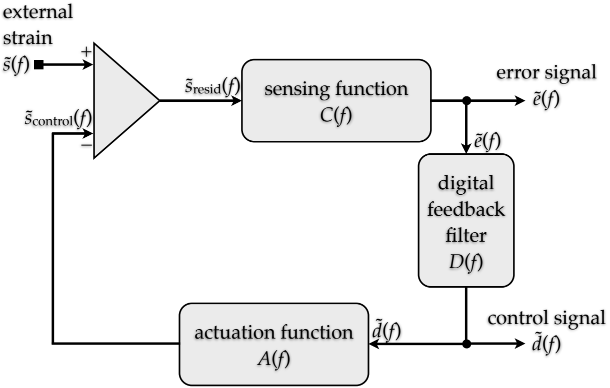
Lengths tuned to keep antisym port dark; power recycling mirror recovers light & sends it back into IFO

Advanced Detectors: Signal Recycling



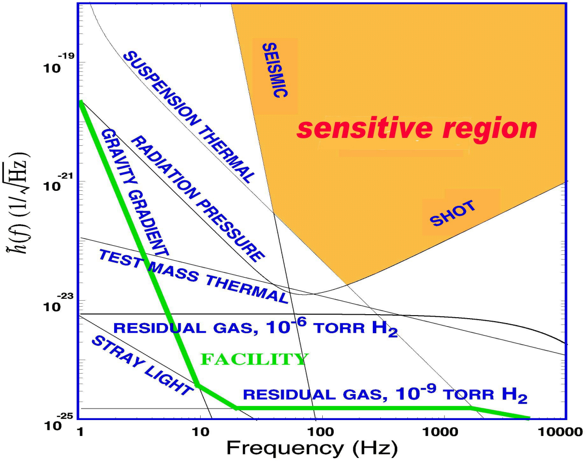
Advanced LIGO/Virgo will also have signal recycling mirror (technology tested by GEO) to decouple noise sources

Sensing, Feedback and Calibration

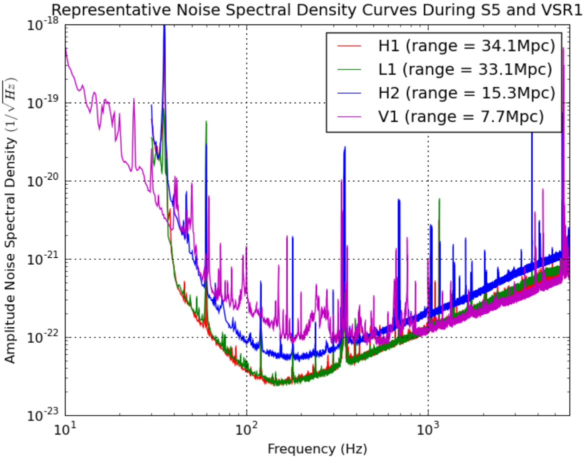


Have to keep FP cavities locked; don't literally let mirrors move in response to GW (& environment); feedback loop keeps IFO in resonance; "GW channel" derived from applied control signal

Sources of Noise in Initial LIGO

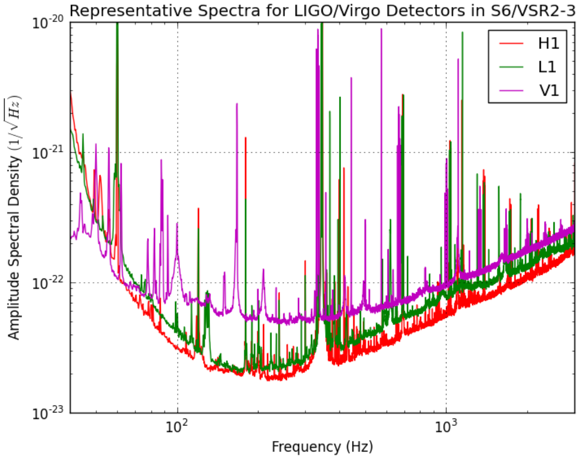


Initial Detector Sensitivities



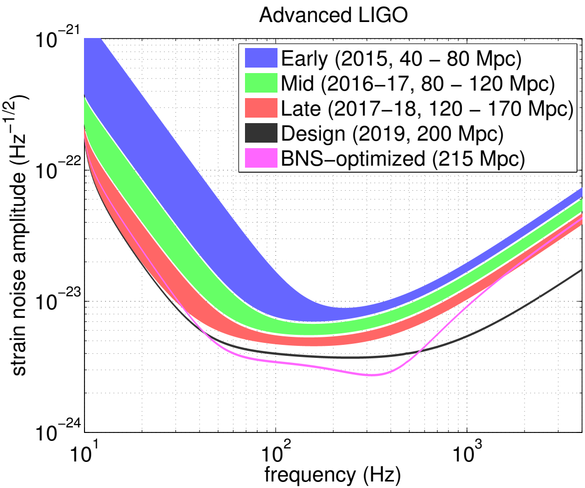
See arXiv:1003.2481

“Enhanced” Detector Sensitivities



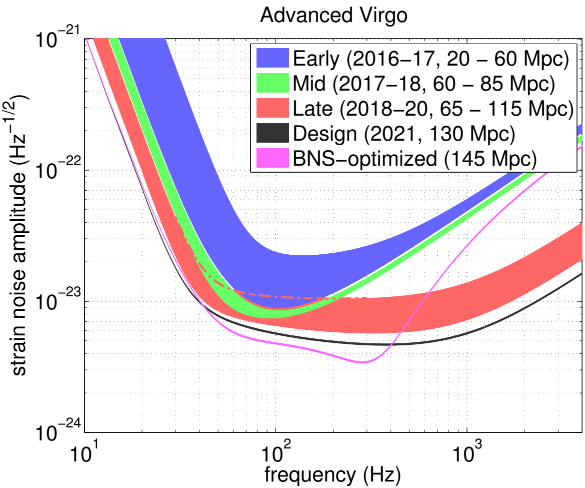
See arXiv:1203.2674

Advanced Detector Expectations



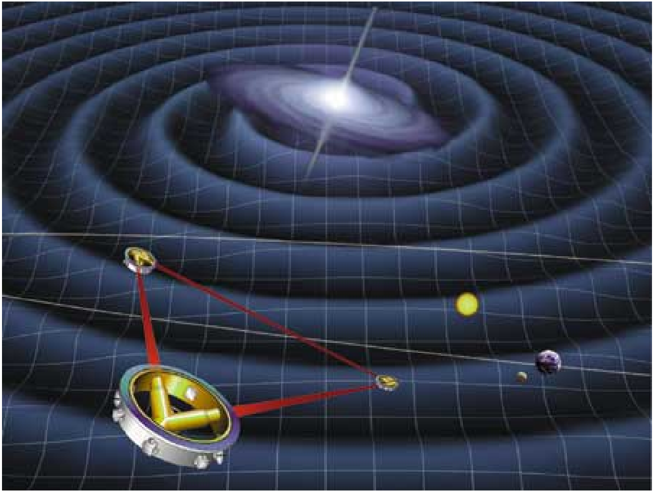
See arXiv:1304.0670

Advanced Detector Expectations



See arXiv:1304.0670

The Saga of Space-Based GW Detectors



The Saga of Space-Based GW Detectors

- LISA (Laser Interferometer Space Antenna) originally proposed in 1993 for 2011 launch; designed to detect mHz GWs from SMBH, galactic WD binaries, EMRIs, etc
- Planned as joint NASA/ESA mission
- Never got funding wedge; dropped by NASA in 2011
- ESA considered “NGO” (LISA-lite) for L-class mission; recently opted for JUICE (moons of Jupiter mission)
- LISA/NGO consistently rated high on science by NASA/ESA, but concerns about practicalities
- LISA Pathfinder Mission flies 2014, to demonstrate technology
- Next ESA L-class misson will be selected in 2015; could fly mid-2020s

Outline

Classification of GW Sources

At freqs relevant to ground-based detectors (10s-1000s of Hz), natural division of sources according to nature of signal

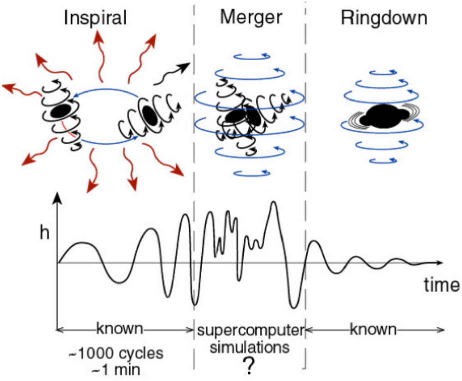
	modelled	unmodelled
long	Periodic Sources (e.g., Rotating Neutron Star)	Stochastic Background (Cosmological or Astrophysical)
short	Binary Coälescence (Black Holes, Neutron Stars)	Bursts (Supernova, short BH Merger, etc.)

Data Analysis Techniques

- Periodic: Waveform well-modelled & long-lived
  Sky position via Doppler modulation
- Stochastic: Cross-correlate detector outputs
  $\rightarrow$ Signal-to-noise improves with time
- Bursts: Signal unmodelled
  $\rightarrow$ Look for unusual features & coherence btwn detecors
  Recent searches incl GRB triggers
- Inspiral: Signal well modelled (at least early)
  $\rightarrow$ Matched Filtering

Template Waveforms for Binary Coalescence

- Inspiralling binaries produce well-modelled GW signals; Search with pattern-match filter
- Compact object binary coälescence consists of inspiral / plunge / merger / ringdown



Cartoon by Kip Thorne

Template Waveforms for Binary Coalescence

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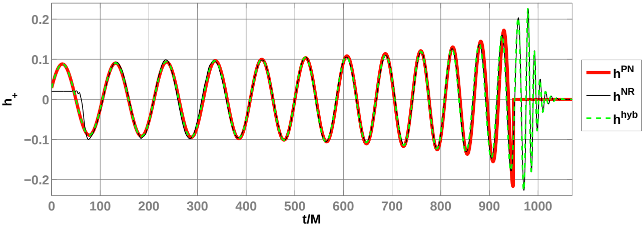


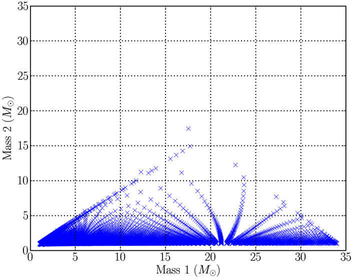
Ajith et al, *CQG* **24**, S689 (2007)

Template Waveforms for Binary Coalescence

- Compact object binary coälescence consists of inspiral / plunge / merger / ringdown
- For first part of inspiral, orbits not too relativistic can expand in powers of $\frac{v}{c} \longrightarrow$ post-Newtonian methods
  Can estimate orb vel from Kepler's 3rd law: $v \approx (\pi G M f)^{1/3}$
  - Low Mass $\longrightarrow$ plunge @ high freq
    $1.4 M_\odot / 1.4 M_\odot$ NS/NS binary has $v \approx 0.3c$ @ 800 Hz;
    PN OK in LIGO band
  - High Mass $\longrightarrow$ plunge @ low freq
    $10 M_\odot / 10 M_\odot$ BH/BH binary has $v \approx 0.4c$ @ 200 Hz;
    merges in LIGO band
- Different template families used for different mass ranges

Matched Filtering GW Data

- Match-filtered signal-to-noise ratio measures how well template "fits" data: $\rho \sim \int df \frac{x^*(f)h(f)}{S_n(f)}$
- Time series for each set of param (e.g., m_1 & m_2) values
- Lay out parameter choices in template bank to get good overlap w/possible signals



Continuous Waves: Searching for Known Pulsars

- Phase params (rotation, sky pos [& binary params]) known
  Pulsar ephemerides (timing) detail phase evolution
- Can search over amplitude params (h_0, ι, ψ, ϕ_0);
  search cost NOT driven by observing time
- Different options for amplitude parameters:
  - Maximize likelihood analytically ($\mathcal{F}$-statistic)
  - Marginalize likelihood numerically ($\mathcal{B}$-statistic)
  - Get posterior prob distribution w/Markov-Chain Monte Carlo
  - Use astro observations to constrain spin orientation (ι & ψ)
- Spindown produces indirect upper limit
  - GW emission above limit $\longrightarrow$ more spindown than seen
  - Pulsars w/rapid spindown have “more room” for GW
  - LIGO/Virgo have surpassed spindown limit for Crab & Vela

Gravitational Waves from Low-Mass X-Ray Binaries

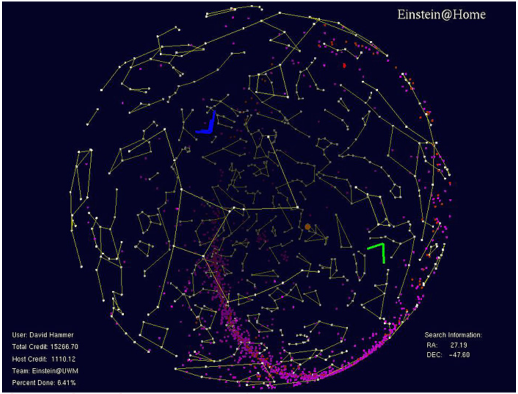
- LMXB: compact object (neutron star or black hole) in binary orbit w/companion star
- If NS, accretion from companion provides "hot spot"; rotating non-axisymmetric NS emits gravitational waves
- Bildsten *ApJL* **501**, L89 (1998) suggested GW spindown may balance accretion spinup; GW strength can be estimated from X-ray flux
- Torque balance would give ≈ constant GW freq
- Signal at solar system modulated by binary orbit

Brightest LMXB: Scorpius X-1

- Scorpius X-1
  - $1.4 M_\odot$ NS w/$0.4 M_\odot$ companion
  - unknown params are f_0, $a \sin i$, orbital phase
- LSC/Virgo searches for Sco X-1:
  - Coherent $\mathcal{F}$-stat search w/6 hr of S2 data
    Abbott et al (LSC) *PRD* **76**, 082001 (2007)
  - Directed stochastic ("radiometer") search (unmodelled)
    Abbott et al (LSC) *PRD* **76**, 082003 (2007)
    Abbott et al (LSC) *PRD* **107**, 271102 (2011)
- Proposed directed search methods:
  - Look for comb of lines produced by orbital modulation
    Messenger & Woan, *CQG* **24**, 469 (2007)
  - Cross-correlation specialized to periodic signal
    Dhurandhar et al *PRD* **77**, 082001 (2008)
- Promising source for Advanced Detectors

Searching for Unknown NSs: Einstein@Home

Semicoherent methods needed to handle phase param space;
Increase computing resources by enlisting volunteers
Distributed using BOINC & run as screensaver



http://www.einsteinathome.org/

Searching for a Stochastic Background

- Noisy data from GW Detector:
  $x(t) = n(t) + h(t) = n(t) + \overleftrightarrow{h}(t) : \overleftrightarrow{d}$
- Look for correlations between detectors

$$\langle x_1 x_2 \rangle = \overbrace{\langle n_1 n_2 \rangle}^{\text{avgto0}} + \overbrace{\langle n_1 h_2 \rangle}^{\text{avgto0}} + \overbrace{\langle h_1 n_2 \rangle}^{\text{avgto0}} + \langle h_1 h_2 \rangle$$

- Expected cross-correlation (frequency domain)

$$\langle \tilde{x}_1^*(f) \tilde{x}_2(f') \rangle = \langle \tilde{h}_1^*(f) \tilde{h}_2(f') \rangle = \overleftrightarrow{d}_1 : \langle \overleftrightarrow{\tilde{h}}{}_1^*(f) \otimes \overleftrightarrow{\tilde{h}}_2(f') \rangle : \overleftrightarrow{d}_2$$

- For stochastic backgrounds

$$\langle \tilde{h}_1^*(f) \tilde{h}_2(f') \rangle = \delta(f - f') \gamma_{12}(f) \frac{S_{\mathrm{gw}}(f)}{2}$$

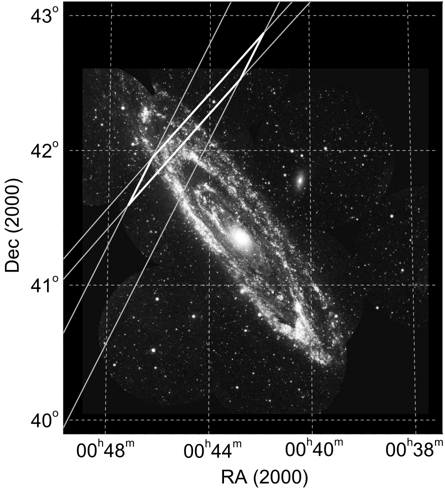
$S_{\mathrm{gw}}(f)$ encodes spectrum; $\gamma_{12}(f)$ encodes geometry

Initial LIGO/Virgo Highlights

- GRB070201 (and GRB051103)
- Crab and Vela spindown
- BBN bound
- Blind Injections

GRB070201

- 2007 Feb 1: short GRB whose error box overlapped spiral arm of M31 (770 kpc away)
- LHO 4 km & 2 km detectors operating & sensitive to CBC out to 35.7& 15.3 Mpc
- No GW seen; rule out CBC progenitor in M31 w/$>$ 99% conf
- *ApJ* **681**, 1419 (2008)



Similar result for GRB051103 & M81; *ApJ* **755**, 2 (2012)

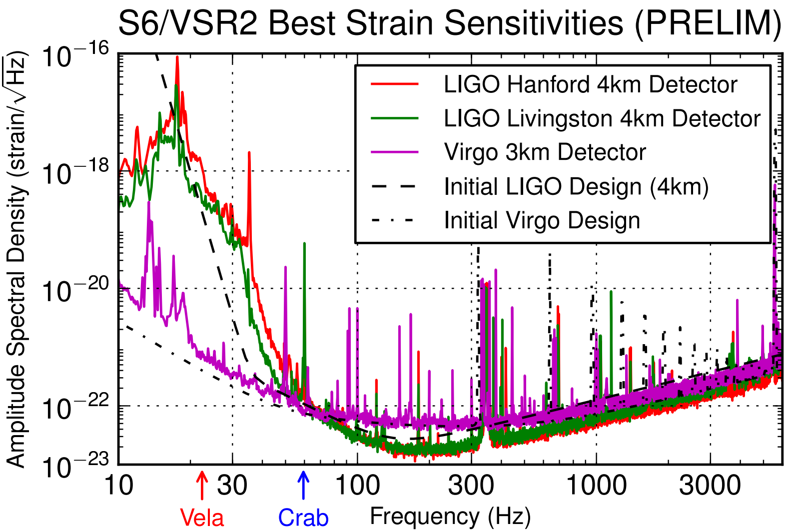
Crab Pulsar Upper Limit

- Pulsar in Crab Nebula
- Created by SN 1054
- $\sim 2\,\text{kpc}$ away
- $f_{\text{rot}} = 29.7\,\text{Hz}$
- $f_{\text{gw}} = 59.4\,\text{Hz}$

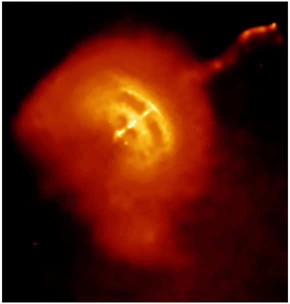
Image credit: Hubble/Chandra

- Initial LIGO (S5) upper limit beats spindown limit
- Abbott et al (LSC) *ApJL* **683**, L45 (2008)
- Abbott et al (LSC & Virgo) + Bégin et al *ApJ* **713**, 671 (2010)
- No more than 2% of spindown energy loss can be in GW

Initial Virgo Targets the Vela Pulsar



Vela Pulsar Upper Limit



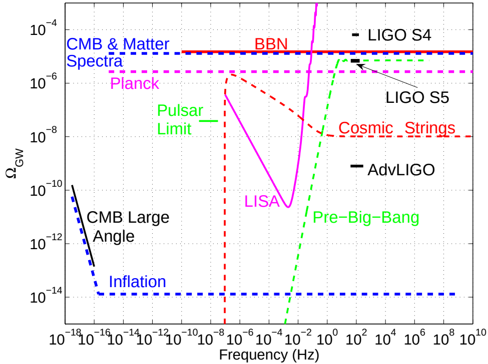
- Pulsar in Vela SN remnant
- Created $\sim$ 12,000 years ago
- $\sim$ 300 pc away
- $f_{\mathrm{rot}} = 11.2\,\mathrm{Hz}$
- $f_{\mathrm{gw}} = 22.4\,\mathrm{Hz}$

Image credit: Chandra

- GW frequency below initial LIGO "seismic wall"
- Virgo has better low-frequency sensitivity
- VSR2 upper limit beats spindown limit
- No more than 10% of spindown energy loss can be in GW

Abadie et al (LSC & Virgo) + Buchner et al *ApJ* **737**, 93 (2011)

Isotropic Stochastic Background Limit



S5 limit $\Omega_{gw}(f) < 6.9 \times 10^{-6} \left(\frac{72\,\mathrm{km/s/Mpc}}{H_0} \right)^2$

[Abbott et al (LSC & Virgo) *Nature* **460**, 990 (2009)]

surpasses indirect limit from Big-Bang Nucleosynthesis

Enhanced LIGO Recovers “Blind Injection”

http://www.ligo.org/science/GW100916/

Summary

- Gravitational waves predicted by GR; energetic but couple weakly to matter
- Generated by rapidly changing mass quadrupole moments, e.g., compact object binaries, rotating NSs, supernovae . . .
- Current state-of-the-art GW detectors: ground-based interferometers, sensitive at $10^1 - 10^3$ Hz. Initial detectors have set upper limits; advanced detectors should make detections
- Ground-based detectors part of GW spectrum analogous to EM spectrum; multi-wavelength GW observations include space-based detectors (planned, $10^{-3} - 10^{-1}$ Hz) & pulsar timing arrays (operating, $10^{-9} - 10^{-7}$ Hz