Gravitational-Wave Astronomy

1060-711: Astronomical Observational Techniques and Instrumentation

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Outline

1 Gravitational-Wave Physics
   - Physical Motivation
   - Mathematical Description
   - Generation of Gravitational Waves

2 Gravitational-Wave Detectors
   - Overview
   - Details of Ground-Based Interferometers
   - Prospects for Space-Based Interferometers

3 Gravitational-Wave Astronomy
   - Gravitational Wave Sources
   - Gravitational Wave Data Analysis
   - Selected Results from First-Generation GW Detectors
Action at a Distance

- Newtonian gravity: mass generates gravitational field
- Lines of force point towards object
Issues with Causality

- Move object; Newton says: lines point to new location
- Relativity says: can’t communicate faster than light to avoid paradoxes
- You could send me supraluminal messages via grav field
If I’m 10 light years away, I can’t know you moved the object 6 years ago.

Far away, gravitational field lines have to point to old location of the object.
Sudden motion (acceleration) of object generates gravitational shock wave expanding at speed of light
Ripples in the Gravitational Field

- Move object back & forth → gravitational wave
- Same argument applies to electricity:
  - can derive magnetism as relativistic effect
  - accelerating charges generate electromagnetic waves propagating @ speed of light
Gravitational Wave from Orbiting Mass?

- Move around in a circle
- Still get grav wave pattern, but looks a bit funny
- Time to move beyond simple pseudo-Newtonian picture
In *Newtonian gravity*, force dep on distance btwn objects
If massive object suddenly moved, grav field at a distance would change *instantaneously*
In relativity, *no* signal can travel faster than light → time-dep grav fields must propagate like light waves
Gravity as Geometry

- **Minkowski Spacetime:**

\[ ds^2 = -c^2(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2 \]

\[
\begin{pmatrix}
    dt \\
    dx \\
    dy \\
    dz
\end{pmatrix}^T
\begin{pmatrix}
    -c^2 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    dt \\
    dx \\
    dy \\
    dz
\end{pmatrix}
= \eta_{\mu\nu} dx^\mu dx^\nu
\]

- **General Spacetime:**

\[ ds^2 = \begin{pmatrix}
    dx^0 \\
    dx^1 \\
    dx^2 \\
    dx^3
\end{pmatrix}^T
\begin{pmatrix}
    g_{00} & g_{01} & g_{02} & g_{03} \\
    g_{10} & g_{11} & g_{12} & g_{13} \\
    g_{20} & g_{21} & g_{22} & g_{23} \\
    g_{30} & g_{31} & g_{32} & g_{33}
\end{pmatrix}
\begin{pmatrix}
    dx^0 \\
    dx^1 \\
    dx^2 \\
    dx^3
\end{pmatrix}
= g_{\mu\nu} dx^\mu dx^\nu
\]
For GW propagation & detection, work to 1st order in $h_{\mu\nu} \equiv$ difference btwn actual metric $g_{\mu\nu}$ & flat metric $\eta_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

($h_{\mu\nu}$ “small” in weak-field regime, e.g. for GW detection)

Convenient choice of gauge is transverse-traceless:

$$h_{0\mu} = h_{\mu0} = 0 \quad \eta^{\nu\lambda} \frac{\partial h_{\mu\nu}}{\partial x^\lambda} = 0 \quad \eta^{\mu\nu} h_{\mu\nu} = \delta^{ij} h_{ij} = 0$$

In this gauge:
- Test particles w/constant coörds are freely falling
- Vacuum Einstein eqns $\rightarrow$ wave equation for $\{h_{ij}\}$:

$$\left(-\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \nabla^2 \right) h_{ij} = 0$$
Far from source, GW looks like plane wave prop along $\vec{k}$

TT conditions mean, in convenient basis,

$$\{k_i\} \equiv k = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \{h_{ij}\} \equiv h = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where $h_+ \left(t - \frac{x^3}{c}\right)$ and $h_\times \left(t - \frac{x^3}{c}\right)$ are components in “plus” and “cross” polarization states

More generally

$$\vec{h} = h_+ \left(t - \frac{\vec{k} \cdot \vec{r}}{c}\right) \hat{e}_+ + h_\times \left(t - \frac{\vec{k} \cdot \vec{r}}{c}\right) \hat{e}_\times$$
The Polarization Basis

wave propagating along $\vec{k}$;
construct $\vec{e}_+, \times$ from $\perp$ unit vectors $\vec{l}$ & $\vec{m}$:

$$\vec{e}_+ = \vec{l} \otimes \vec{l} - \vec{m} \otimes \vec{m} \quad \vec{e}_\times = \vec{l} \otimes \vec{m} + \vec{m} \otimes \vec{l}$$
Fluctuating geom changes distances btwn particles in free-fall:

<table>
<thead>
<tr>
<th>Plus (+) Polarization</th>
<th>Cross (×) Polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Circle" /></td>
<td><img src="image2" alt="Circle" /></td>
</tr>
</tbody>
</table>
**Multipole Expansion for Gravitational Radiation**

- **“Electric Dipole”**? No, “dipole moment” \( \int \vec{r} \, dm \propto \text{ctr of mass} \)
  - COM can’t oscillate (also no negative “charge” in GR)
- **“Magnetic Dipole”**? No, “mag moment”
  - \( \frac{1}{2} \int \vec{r} \times \vec{v} \, dm \propto \text{spin}, \) another conserved quantity
- **“Electric Quadrupole”**? Yes! In TT gauge,

\[
 h_{ij}(t) = \frac{2G}{c^4 d} P^\text{TT} k_k \ell_i j_k \ell(t - \frac{d}{c})
\]

in terms of mass quadrupole moment

\[
 +_{ij} = \int \left( r_i r_j - \delta_{ij} \frac{r^2}{3} \right) \, dm
\]
Quadrupole Radiation From Rotating/Orbiting System

- Equatorial moments of inertia $I_1, I_2$
- Orbital/rotational angular vel $\Omega$
- GW frequency $f_{gw} = \frac{\Omega}{2\pi}$

Since $\dot{I} \sim (2\Omega)^2 |I_1 - I_2|$, 

$$\leftrightarrow h = \frac{4G\Omega^2(I_1 - I_2)}{c^4 d} \left( \leftrightarrow e_+ \frac{1 + \cos^2 \iota}{2} \cos 2\Omega t + \leftrightarrow e_\times \cos \iota \sin 2\Omega t \right)$$

- For binary system w/masses $m_1, m_2$ and separation $r$, 

  $$I_1 = 0 \quad \text{and} \quad I_2 = \mu r^2$$

  where $\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_1 m_2}{M}$ is the reduced mass
Radiation from Quasicircular Binary

Total mass $M = m_1 + m_2$; reduced mass $\mu = \frac{m_1 m_2}{M}$; orbital freq $\Omega$

- Amplitude is $h_0 = \frac{4G\Omega^2 \mu r^2}{c^4 d}$

- Kepler’s 3rd law: $GM = r^3 \Omega^2 \implies r^2 = (GM\Omega^{-2})^{2/3}$

$$h_0 = \frac{4G^{5/3} M^{2/3} \mu^{2/3} \Omega^{2/3}}{c^4 d} = \frac{4(Mc^\gamma)^{5/3} \Omega^{2/3}}{c^4 d}$$

where $M_c = M \eta^{3/5}$ is chirp mass & $\eta = \frac{\mu}{M}$ is symm mass ratio

- Orbit will evolve due to GW emission (radiation reaction): energy lost, $r$ dec., $\Omega$ inc., $h_0$ inc.: “chirp”

- Quasicircular assumption breaks down when $r_{isco} \approx 6GM/c^2$ near “innermost stable circular orbit” (ISCO); orbital freq @ ISCO is $\Omega_{isco} \approx \sqrt{\frac{GM}{r_{isco}^3}} = \frac{c^3}{6^{3/2}GM}$

- Modelling final merger accurately requires numerical simulations like those done in RIT CCRG
Some Sources of Gravitational Waves

Band: ground, space, pulsar timing

- Binary coalescence (inspiral+merger+ringdown):
  - Supermassive BH binary
  - extreme mass ratio (stellar mass + SMBH)
  - Stellar mass BH and/or neutron star

- Galactic white dwarf binary orbit (continuous source)

- Rotating neutron star (pulsar, LMXB, etc)

- Supernova, SGR

- Cosmological background
  (primordial, phase transitions, cosmic superstrings, etc)

- SMBH flyby
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Methods for Measuring Gravitational Waves

- Cosmic Microwave Background Perturbations
  \( f_{gw} \sim H_0 \sim 10^{-18} \text{ Hz} \)
- Pulsar Timing Arrays \((10^{-9} \text{ Hz} \lesssim f_{gw} \lesssim 10^{-7} \text{ Hz})\)
- Laser Interferometers
  - Space-Based \((10^{-3} \text{ Hz} \lesssim f_{gw} \lesssim 10^{-1} \text{ Hz})\)
  - Ground-Based \((10^1 \text{ Hz} \lesssim f_{gw} \lesssim 10^3 \text{ Hz})\)
- Resonant-Mass Detectors (narrowband, \( f_{gw} \sim 10^3 \text{ Hz} \))

Note, observable GW freq cover 20 orders of magnitude, similar to EM radiation, but the frequencies are much lower
\((10^3 \text{ Hz} \lesssim f_{em} \lesssim 10^{23} \text{ Hz})\)
Rogues’ Gallery of Ground-Based Interferometers

LIGO Hanford (Wash.)
GEO-600 (Germany)

LIGO Livingston (La.)
Virgo (Italy)
Initial Gravitational Wave Detector Network

“1st generation” ground-based interferometric GW detectors (kilometer scale):
- TAMA 300 (Tokyo, Japan) first online, late 90s; now offline
- LSC detectors conducting science runs since 2002
  - LIGO Hanford (4km H1 & 2km H2)
  - LIGO Livingston (4km L1)
  - GEO-600 (600m G1)
- Virgo (3km V1) started science runs in 2007
- LSC-Virgo long joint runs @ design sensitivity 2005-2010

LIGO and Virgo being upgraded to 2nd generation “advanced” detectors (10× improvement in sensitivity)

GEO-600 remains operational in “astrowatch” mode in case there’s a nearby supernova
Advanced Gravitational Wave Detector Network

“2nd generation” ground-based interferometric GW detectors:
- Adv LIGO expected to take science data from 2014 or 2015
  4km detectors in Livingston, La. & Hanford, Wa.
- Advanced Virgo should be on comparable timescale
- KAGRA (cryogenic detector in Kamioka mine, Japan) uses 2.5-generation technology
- Third advanced LIGO detector (4km) may be installed in India, taking data c.2019+
  Big payoff for sky localization via tringulation

Planning for 3rd generation already underway:
- Einstein Telescope in Europe
- USA 3G plans still under development
  (RIT CCRG involved in astrophysics planning)
Experimental Details: LIGO

- Initial/enhanced LIGO was a power-recycled Fabry-Pérot Michelson interferometer
- Advanced LIGO will be a dual-recycled Fabry-Pérot Michelson interferometer
- Basic idea: use interferometry to measure changes in difference of arm lengths to detect $h \lesssim 10^{-20}$
Michelson Interferometer

\[
\text{w/ } \lambda_{\text{laser}} \sim 10^{-6} \text{ m } \& \text{ } L \sim 10^3 \text{ m}
\]

would need to measure

\[
\delta L \sim 10^{-11} \lambda_{\text{laser}}
\]

to detect \( h \sim 10^{-20} \)
Fabry-Pérot Cavities

Increase “effective length” of arms by keeping light in resonance within FP cavities; finesse $\sim 200$ amplifies signal
Power Recycling

Lengths tuned to keep antisym port dark; power recycling mirror recovers light & sends it back into IFO
Advanced Detectors: Signal Recycling

Advanced LIGO/Virgo will also have signal recycling mirror (technology tested by GEO) to decouple noise sources
Have to keep FP cavities locked; don’t literally let mirrors move in response to GW (& environment); feedback loop keeps IFO in resonance; “GW channel” derived from applied control signal
Sources of Noise in Initial LIGO

![Graph showing various noise sources in gravitational wave detection](image-url)
Initial Detector Sensitivities

Representative Noise Spectral Density Curves During S5 and VSR1

- H1 (range = 34.1Mpc)
- L1 (range = 33.1Mpc)
- H2 (range = 15.3Mpc)
- V1 (range = 7.7Mpc)

See arXiv:1003.2481
See arXiv:1203.2674
Advanced Detector Expectations

Advanced LIGO

- Early (2015, 40 – 80 Mpc)
- Mid (2016–17, 80 – 120 Mpc)
- Late (2017–18, 120 – 170 Mpc)
- Design (2019, 200 Mpc)
- BNS–optimized (215 Mpc)

See arXiv:1304.0670
Advanced Detector Expectations

Advanced Virgo

- Early (2016−17, 20 − 60 Mpc)
- Mid (2017−18, 60 − 85 Mpc)
- Late (2018−20, 65 − 115 Mpc)
- Design (2021, 130 Mpc)
- BNS−optimized (145 Mpc)

See arXiv:1304.0670
The Saga of Space-Based GW Detectors
The Saga of Space-Based GW Detectors

- **LISA** (Laser Interferometer Space Antenna) originally proposed in 1993 for 2011 launch; designed to detect mHz GWs from SMBH, galactic WD binaries, EMRIs, etc.
- Planned as joint NASA/ESA mission.
- Never got funding wedge; dropped by NASA in 2011.
- ESA considered “NGO” (LISA-lite) for L-class mission; recently opted for JUICE (moons of Jupiter mission).
- LISA/NGO consistently rated high on science by NASA/ESA, but concerns about practicalities.
- LISA Pathfinder Mission flies 2014, to demonstrate technology.
- Next ESA L-class mission will be selected in 2015; could fly mid-2020s.
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### Classification of GW Sources

At freqs relevant to ground-based detectors (10s-1000s of Hz), natural division of sources according to nature of signal

<table>
<thead>
<tr>
<th></th>
<th>modelled</th>
<th>unmodelled</th>
</tr>
</thead>
<tbody>
<tr>
<td>long</td>
<td><strong>Periodic Sources</strong> (e.g., Rotating Neutron Star)</td>
<td><strong>Stochastic Background</strong> (Cosmological or Astrophysical)</td>
</tr>
<tr>
<td>short</td>
<td><strong>Binary Coalescence</strong> (Black Holes, Neutron Stars)</td>
<td><strong>Bursts</strong> (Supernova, short BH Merger, etc.)</td>
</tr>
</tbody>
</table>
Data Analysis Techniques

- **Periodic**: Waveform well-modelled & long-lived
  Sky position via Doppler modulation

- **Stochastic**: Cross-correlate detector outputs
  → Signal-to-noise improves with time

- **Bursts**: Signal unmodelled
  → Look for unusual features & *coherence* btwn detectors
  Recent searches incl GRB triggers

- **Inspiral**: Signal well modelled (at least early)
  → Matched Filtering
Template Waveforms for Binary Coalescence

- Inspiralling binaries produce well-modelled GW signals; Search with pattern-match filter
- Compact object binary coalescence consists of inspiral / plunge / merger / ringdown

Cartoon by Kip Thorne
Template Waveforms for Binary Coalescence

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- Compact object binary coalescence consists of inspiral / plunge / merger / ringdown

Template Waveforms for Binary Coalescence

- Compact object binary coalescence consists of inspiral / plunge / merger / ringdown
- For first part of inspiral, orbits not too relativistic can expand in powers of $\frac{v}{c} \rightarrow$ post-Newtonian methods
- Can estimate orb vel from Kepler’s 3rd law: $v \approx (\pi GMf)^{1/3}$
  - Low Mass $\rightarrow$ plunge @ high freq
    - $1.4M_\odot/1.4M_\odot$ NS/NS binary has $v \approx 0.3c$ @ 800 Hz;
    - PN OK in LIGO band
  - High Mass $\rightarrow$ plunge @ low freq
    - $10M_\odot/10M_\odot$ BH/BH binary has $v \approx 0.4c$ @ 200 Hz;
    - merges in LIGO band
- Different template families used for different mass ranges
Matched Filtering GW Data

- Match-filtered signal-to-noise ratio measures how well template “fits” data: $\rho \sim \int df \frac{x^*(f)h(f)}{S_n(f)}$
- Time series for each set of param (e.g., $m_1$ & $m_2$) values
- Lay out parameter choices in template bank to get good overlap w/possible signals
Continuous Waves: Searching for Known Pulsars

- **Phase params** (rotation, sky pos [& binary params]) known
  Pulsar ephemerides (timing) detail phase evolution

- Can search over **amplitude params** \((h_0, \nu, \psi, \phi_0)\);
  search cost **NOT** driven by observing time

- Different options for **amplitude parameters**:
  - Maximize likelihood analytically (\(F\)-statistic)
  - Marginalize likelihood numerically (\(B\)-statistic)
  - Get posterior prob distribution w/Markov-Chain Monte Carlo
  - Use astro observations to constrain spin orientation (\(\nu\) & \(\psi\))

- Spindown produces **indirect upper limit**
  - GW emission above limit \(\rightarrow\) more spindown than seen
  - Pulsars w/rapid spindown have “more room” for GW
  - LIGO/Virgo have surpassed spindown limit for Crab & Vela
LMXB: compact object (neutron star or black hole) in binary orbit with companion star
- If NS, accretion from companion provides “hot spot”; rotating non-axisymmetric NS emits gravitational waves
- Bildsten *ApJL* 501, L89 (1998) suggested GW spindown may balance accretion spinup; GW strength can be estimated from X-ray flux
- Torque balance would give ≈ constant GW freq
- Signal at solar system modulated by binary orbit
Brightest LMXB: Scorpius X-1

- Scorpius X-1
  - $1.4M_\odot$ NS w/ $0.4M_\odot$ companion
  - unknown params are $f_0$, $a \sin i$, orbital phase
- LSC/Virgo searches for Sco X-1:
  - Coherent $\mathcal{F}$-stat search w/ 6 hr of S2 data
    Abbott et al (LSC) *PRD* 76, 082001 (2007)
  - Directed stochastic ("radiometer") search (unmodelled)
    Abbott et al (LSC) *PRD* 76, 082003 (2007)
    Abbott et al (LSC) *PRD* 107, 271102 (2011)
- Proposed directed search methods:
  - Look for comb of lines produced by orbital modulation
    Messenger & Woan, *CQG* 24, 469 (2007)
  - Cross-correlation specialized to periodic signal
    Dhurandhar et al *PRD* 77, 082001 (2008)
- Promising source for Advanced Detectors
Searching for Unknown NSs: Einstein@Home

Semicohherent methods needed to handle phase param space; increase computing resources by enlisting volunteers. Distributed using BOINC & run as screensaver.

http://www.einsteinathome.org/
Searching for a Stochastic Background

- Noisy data from GW Detector:
  \[ x(t) = n(t) + h(t) = n(t) + \dot{h}(t) : d \]

- Look for correlations between detectors
  \[
  \langle x_1 x_2 \rangle = \langle n_1 n_2 \rangle + \langle n_1 h_2 \rangle + \langle h_1 n_2 \rangle + \langle h_1 h_2 \rangle
  \]

- Expected cross-correlation (frequency domain)
  \[
  \langle \tilde{x}_1^*(f) \tilde{x}_2(f') \rangle = \langle \tilde{h}_1^*(f) \tilde{h}_2(f') \rangle = d_1 : \langle \tilde{h}_1^*(f) \otimes \tilde{h}_2(f') \rangle : d_2
  \]

- For stochastic backgrounds
  \[
  \langle \tilde{h}_1^*(f) \tilde{h}_2(f') \rangle = \delta(f - f') \gamma_{12}(f) \frac{S_{gw}(f)}{2}
  \]

\[ S_{gw}(f) \text{ encodes spectrum; } \gamma_{12}(f) \text{ encodes geometry} \]
Initial LIGO/Virgo Highlights

- GRB070201 (and GRB051103)
- Crab and Vela spindown
- BBN bound
- Blind Injections
GRB070201

- 2007 Feb 1: short GRB whose error box overlapped spiral arm of M31 (770 kpc away)
- LHO 4 km & 2 km detectors operating & sensitive to CBC out to 35.7 & 15.3 Mpc
- No GW seen; rule out CBC progenitor in M31 w/ > 99% conf

Crab Pulsar Upper Limit

- Pulsar in Crab Nebula
- Created by SN 1054
- $\sim 2$ kpc away
- $f_{\text{rot}} = 29.7$ Hz
- $f_{\text{gw}} = 59.4$ Hz

Image credit: Hubble/Chandra

- Initial LIGO (S5) upper limit beats spindown limit
- No more than 2% of spindown energy loss can be in GW
Initial Virgo Targets the Vela Pulsar

S6/VSR2 Best Strain Sensitivities (PRELIM)

Amplitude Spectral Density (strain/\sqrt{Hz})

- LIGO Hanford 4km Detector
- LIGO Livingston 4km Detector
- Virgo 3km Detector
- Initial LIGO Design (4km)
- Initial Virgo Design

Frequency (Hz)

Vela 30 100 300 1000 3000
Crab
Vela Pulsar Upper Limit

- Pulsar in Vela SN remnant
- Created $\sim 12,000$ years ago
- $\sim 300$ pc away
- $f_{\text{rot}} = 11.2$ Hz
- $f_{\text{gw}} = 22.4$ Hz

Image credit: Chandra

- GW frequency below initial LIGO "seismic wall"
- Virgo has better low-frequency sensitivity
- VSR2 upper limit beats spindown limit
- No more than 10% of spindown energy loss can be in GW

S5 limit $\Omega_{gw}(f) < 6.9 \times 10^{-6} \left( \frac{72 \text{ km/s/Mpc}}{H_0} \right)^2$

Enhanced LIGO Recovers “Blind Injection”

http://www.ligo.org/science/GW100916/
Gravitational waves predicted by GR; energetic but couple weakly to matter

Generated by rapidly changing mass quadrupole moments, e.g., compact object binaries, rotating NSs, supernovae . . .

Current state-of-the-art GW detectors: ground-based interferometers, sensitive at $10^1 - 10^3$ Hz. Initial detectors have set upper limits; advanced detectors should make detections

Ground-based detectors part of GW spectrum analogous to EM spectrum; multi-wavelength GW observations include space-based detectors (planned, $10^{-3} - 10^{-1}$ Hz) & pulsar timing arrays (operating, $10^{-9} - 10^{-7}$ Hz)