Frequency bins for quantum information processing

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Time-frequency QIP

- Several QIP protocols explored, distinguished by
  - **Encoding:** discrete/continuous.
  - **Photonic mode:** time-bin/frequency-bin/pulsed.
  - **Processing:** Nonlinear mixing/linear optics.

Quantum Pulse Gate

Frequency-Bin CV Cluster States


Time-Bin LOQC


**Spectral LOQC**


The first discrete, linear-optical QIP protocol for frequency-bin qubits.

**LOQC:** Linear-optical quantum computation

Why frequency bins?

- Quantum information encoded in photon frequency/wavelength.
  - Compatible with classical telecom.
  - Relies on optical fiber.
  - Applicable to on-chip quantum light sources.
  - Useful for connecting qubits in quantum internet.


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Standard Approach:
must have $\omega_A = \omega_B$

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Our Idea:
use $\omega_A \neq \omega_B$ for encoding


Key technology 1: Fourier-transform pulse shaping

Pulse Doublet

Dispersion Compensation

Comb-less Pulse Train

Frequency Encoding

CLASSICAL EXAMPLES


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**Pulse Doublet**

**Dispersion Compensation**

**QUANTUM EXAMPLES**

4f Pulse Shaper


**Comb-less Pulse Train**

**Frequency Encoding**


Key technology 2: electro-optic modulation

Modulation Cancellation

Tunable Delay

CLASSICAL EXAMPLES

Electro-Optic Phase Modulator

Spread Spectrum

Time Lens


Key technology 2: electro-optic modulation

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QUANTUM EXAMPLES

Eletro-Optic Phase Modulator


Time Lens


Universal QIP with frequency-bin qubits

1. **Qubit**: One photon, two spectral bins.

   \[ |\psi\rangle = \alpha |0\rangle_L + \beta |1\rangle_L \]
   \[ = (\alpha \hat{a}^\dagger_0 + \beta \hat{a}^\dagger_1) |\text{vac}\rangle \]

2. **Phase shifter**: Fourier-transform pulse shaper.

   \[ \hat{b}_n = e^{i\phi_n} \hat{a}_n \]

3. **Mode mixer**: Electro-optic phase modulator (EOM).

   \[ e^{i\varphi(t)} = \sum_k c_k e^{-ik\Delta\omega t} \]
   \[ \hat{b}_n = \sum_k c_{n-k} \hat{a}_k \]
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\[ e^{i\varphi(t)} = \sum_k c_{n-k} \omega_k \]

**Our Findings**

These elements are sufficient for universal, scalable quantum information processing.

**Theory Paper**

Quantum frequency processor (QFP)

- Our experiments so far have concentrated on a quantum frequency processor (QFP) with
  
  i. Three elements (EOM–PS–EOM).
  
  ii. Sinewave-only EO modulation.

Basic QFP

- Enables near-ideal single-qubit gates, and high-fidelity two-qubit gates.
- Characterize with classical frequency comb, using method analogous to [S. Rahimi-Keshari et al., Opt. Express 21, 13450 (2013)].
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Experimental setup at Oak Ridge
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Implemented the frequency-bin beamsplitter (Hadamard $H$ gate) experimentally.

Measured $\mathcal{F} = 0.99998 \pm 0.00003$ & $\mathcal{P} = 0.9739 \pm 0.0003$. Examples:
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Frequency-bin tritter

- Can also extend to $3 \times 3$ system—a tritter.

- To the single-photon level!

Frequency-domain Hong–Ou–Mandel interference

- Requires:
  1. Frequency beamsplitter.
  2. Tunable “distinguishability” parameter.

- Previous examples:
  - $\chi^{(2)}$ mixer, scan delay
    $$\gamma = 0.71 \pm 0.04$$
    T. Kobayashi et al., Nat. Photon. 10, 441 (2016).
  - $\chi^{(3)}$ mixer, scan frequency spacing
    $$\gamma = 0.68 \pm 0.03$$
  - EO mixer, scan frequency spacing
    $$\gamma = 0.84 \pm 0.02$$
    (accidentals subtracted)

Our approach

We tune BS reflectivity $R$ by scanning pulse shaper phase shift $\alpha$. 
Our frequency-bin HOM interferometer

- Filter out central modes: $|\Psi\rangle = |\omega_0\rangle_A |\omega_1\rangle_B$.
- Scan $\alpha$ for tunable BS between $\omega_0$ and $\omega_1$.
- Coincidences: $C_{01} \propto |R(\alpha) - T(\alpha)|^2$.

Findings

1. Record-high visibility for frequency HOM: $V = 0.97 \pm 0.01$.
2. Minimal scattering to adjacent modes.
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Multi-qubit control

- New aspects demonstrated by our HOM interferometer:
  1. **Tunable reflectivity** ⇒ reconfigurable quantum gates.
  2. **Frequency-based tuning** ⇒ independent gates in parallel in **same configuration**.

Operation on two qubits

We set up two single-qubit gates in parallel, each is either identity \( I (\mathcal{R} = 0) \) or Hadamard \( H (\mathcal{R} = 0.5) \).

- Filtered input: \( |\Psi\rangle = |\omega_{-4}\rangle_A |\omega_5\rangle_B + |\omega_{-3}\rangle_A |\omega_4\rangle_B \)

\[
\begin{align*}
1_A \otimes 1_B & \rightarrow |\omega_{-4}\rangle_A |\omega_5\rangle_B + |\omega_{-3}\rangle_A |\omega_4\rangle_B \\
H_A \otimes 1_B & \rightarrow |\omega_{-4}\rangle_A |\omega_4\rangle_B - |\omega_{-4}\rangle_A |\omega_5\rangle_B - |\omega_{-3}\rangle_A |\omega_4\rangle_B + |\omega_{-3}\rangle_A |\omega_5\rangle_B \\
1_A \otimes H_B & \rightarrow |\omega_{-4}\rangle_A |\omega_3\rangle_B - |\omega_{-4}\rangle_A |\omega_5\rangle_B + |\omega_{-3}\rangle_A |\omega_4\rangle_B + |\omega_{-3}\rangle_A |\omega_5\rangle_B \\
H_A \otimes H_B & \rightarrow |\omega_{-4}\rangle_A |\omega_4\rangle_B + |\omega_{-3}\rangle_A |\omega_5\rangle_B
\end{align*}
\]
Multi-qubit control

- New aspects demonstrated by our HOM interferometer:
  1. Tunable reflectivity $\Rightarrow$ reconfigurable quantum gates.
  2. Frequency-based tuning $\Rightarrow$ independent gates in parallel in same configuration.

Operation on two qubits

We set up two single-qubit gates in parallel, each is either identity $\mathbb{1}$ ($R = 0$) or Hadamard $H$ ($R = 0.5$).

- Filtered input: $|\Psi\rangle = |\omega_{-4}\rangle_A |\omega_5\rangle_B + |\omega_{-3}\rangle_A |\omega_4\rangle_B$

  $\mathbb{1}_A \otimes \mathbb{1}_B \rightarrow |\omega_{-4}\rangle_A |\omega_5\rangle_B + |\omega_{-3}\rangle_A |\omega_4\rangle_B$ $\text{Negative frequency correlations}$

  $H_A \otimes \mathbb{1}_B \rightarrow |\omega_{-4}\rangle_A |\omega_4\rangle_B + |\omega_{-3}\rangle_A |\omega_5\rangle_B$ $\text{No frequency correlations}$

  $\mathbb{1}_A \otimes H_B \rightarrow |\omega_{-4}\rangle_A |\omega_3\rangle_B + |\omega_{-3}\rangle_A |\omega_4\rangle_B + |\omega_{-4}\rangle_A |\omega_5\rangle_B + |\omega_{-3}\rangle_A |\omega_5\rangle_B$ $\text{No frequency correlations}$

  $H_A \otimes H_B \rightarrow |\omega_{-4}\rangle_A |\omega_4\rangle_B + |\omega_{-3}\rangle_A |\omega_5\rangle_B$ $\text{Positive frequency correlations}$
Independent, parallel qubit control

\begin{align*}
&1_A \otimes 1_B \\
&H_A \otimes 1_B \\
&1_A \otimes H_B \\
&H_A \otimes H_B
\end{align*}

Flipped Frequency Correlations!
Independent, parallel qubit control

Conditional entropy

\[ \mathcal{H}(U_A|U_B) = \text{uncertainty in } A\text{’s result given } B\text{’s}. \]

\[ \mathcal{H}(1_A|1_B) = 0.19 \pm 0.03 \]

\[ \mathcal{H}(H_A|1_B) = 0.997 \pm 0.003 \]

Entanglement witness

\[ \mathcal{H}(1_A|1_B) + \mathcal{H}(H_A|H_B) = 0.48 \pm 0.05 < q_{MU} (= 0.9714) \checkmark \]

\[ \mathcal{H}(1_A|H_B) = 0.993 \pm 0.005 \]

\[ \mathcal{H}(H_A|H_B) = 0.29 \pm 0.04 \]
Bayesian state reconstruction

- BME can recover full state from previous four measurements alone.
- Retrieved error accounts for missing information.

\[
\text{Re}(\hat{\rho}) \quad \text{Im}(\hat{\rho})
\]

\[
\sigma\{\text{Re}(\hat{\rho})\} \quad \sigma\{\text{Im}(\hat{\rho})\}
\]

Legend:

- 00 \equiv |1_{\omega-4}A1_{\omega4}B
- 01 \equiv |1_{\omega-4}A1_{\omega5}B
- 10 \equiv |1_{\omega-3}A1_{\omega4}B
- 11 \equiv |1_{\omega-3}A1_{\omega5}B
Bayesian state reconstruction

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\[ F = \langle \Psi^+ | \hat{\rho} | \Psi^+ \rangle = 0.92 \pm 0.01 \]

Legend

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01 \equiv |1_{\omega-4}\rangle_A |1_{\omega5}\rangle_B
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Two-qubit gate

- Entangling gates also necessary for universal QIP.
- Challenging with optics, but possible probabilistically.
- Design and implement coincidence-basis CNOT in our QFP.

Key result

First entangling gate for frequency-encoded qubits, in any platform.
Two-qubit gate

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Back to Bayes

- Conventional reconstruction provides no information on coherence from computational basis alone.

- We develop model and apply *Bayesian machine learning* and slice sampling to analyze the full quantum CNOT.

- Utilizes all data: singles, coincidences, no counts.

$$P(\mathcal{D} | \beta) = (p_A - p_{AB})^{N_A - N_{AB}} (p_B - p_{AB})^{N_B - N_{AB}} \\ \times p_{AB}^{-N_{AB}} (1 - p_A - p_B + p_{AB})^{M - N_A - N_B + N_{AB}}$$

- Obtain quantum unitary fidelity of $F_{\text{Bayes}} = 0.91 \pm 0.01$.

Bayesian machine learning

Extracts details from experimental data hidden from traditional quantum characterization methods.
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Applications for frequency-bin QIP

- Demonstrated universal gate set. What’s on the horizon?
  1. Quantum simulation.
  2. Time-frequency hyperencoding.
  3. On-chip integration for scalability, low loss.
  4. Quantum node connections.
  5. Classical all-optical networking.

Acknowledgments

References